Chapter 10
Polarization Mode Dispersion in Optical Fibers

10.1 Introduction

In the previous chapter, we discussed devices based on deliberately introduced birefringence in an optical fiber, thus utilizing the polarization effects to our advantage. In Chapter 10, we discuss the detrimental aspect of birefringence in optical fibers—namely, polarization mode dispersion (PMD).

The existence of birefringence in a fiber implies that the fiber supports two orthogonally polarized modes that have different effective indices and hence propagate with different group velocities in the fiber. An optical pulse launched into such a fiber would be split into two orthogonally polarized pulses, which would then propagate with different propagation constants and group velocities. The two pulses thus reach the output end of the fiber at slightly different times and with different phases. The superimposition of these two pulses leads to the generation of an optical pulse that is now temporally more broadened as compared to the input pulse. Thus, the pulse becomes dispersed due to the effect of fiber birefringence, and the phenomenon is called polarization mode dispersion. PMD is a serious limitation in the case of ultrahigh-bit-rate (> 10 Gb) fiber communication links, as it puts a cap on the bit rate of the link as well as causes errors in data transmission. In the following, we discuss some basic concepts involved in the understanding of PMD in optical fibers.

10.2 PMD in Short-Length and High-Birefringence Fibers

In short-length SMFs or Hi-Bi fibers, one can assume that the birefringence is constant in magnitude as well as in direction so that there is no polarization mode coupling. In such cases, the two polarization modes—namely, slow and fast—are fixed, so PMD is completely deterministic and not random. If \( n_s \) and \( n_f \) are the effective indices of the slow and fast modes, respectively, the corresponding propagation constants will be given by
\[ \beta_s = \frac{\omega}{c} n_s \quad \text{and} \quad \beta_f = \frac{\omega}{c} n_f, \quad (10.1) \]

where \( \omega \) is the angular frequency. The two orthogonally polarized pulses will be traveling with different group velocities given by

\[ \frac{1}{v_{gs}} = \frac{d\beta_s}{d\omega}, \quad \text{and} \quad \frac{1}{v_{gf}} = \frac{d\beta_f}{d\omega}. \quad (10.2) \]

Thus, the two pulses will take different times to travel a given length \( L \) of the fiber. The difference in the propagation times \( \Delta \tau \) is known as the differential group delay (DGD) of the fiber and is given by

\[ \Delta \tau = \frac{L}{v_{gs}} - \frac{L}{v_{gf}} = L \frac{d}{d\omega} [\beta_s - \beta_f] = L \Delta \beta', \quad (10.3) \]

where \( \Delta \beta = (\beta_s - \beta_f) \), and \( \Delta \beta' \) represents the derivative of \( \Delta \beta \) with respect to \( \omega \). The DGD is taken as the measure of PMD of a given fiber. Figure 10.1 shows the effect of the PMD on the input pulse in the time domain, demonstrating that an input pulse is divided into two orthogonally polarized pulses that are separated in time by \( \Delta \tau \) at the output end.

It should be noted that in short-length fibers, the DGD grows linearly with fiber length [see Eq. (10.3)]. It is clear from Eq. (10.1) that the two polarized modes will accumulate a phase difference between them, which at a distance \( z \) from the input end is given by

\[ \delta(\omega, z) = (\beta_s - \beta_f) z = \frac{\omega}{c} (n_s - n_f) z = \frac{\omega}{c} (\Delta n_{\text{eff}}) z, \quad (10.4) \]

where \( \Delta n_{\text{eff}} = (n_s - n_f) \). The phase difference \( \delta(\omega, z) \) depends on both frequency and distance. As a result, the SOP of a polarized incident pulse will change with both distance and input frequency. Equation (10.3) can also be put in the form

**Figure 10.1** A schematic of pulse broadening due to PMD.
Equation (10.5) shows that $\Delta \tau$ is equal to the rate of change of $\delta(\omega, L)$ with respect to frequency $\omega$. A change in the phase difference $\delta(\omega, L)$ means a change in the output SOP, so it is implied that $\Delta \tau$ can be obtained by knowing the rate of change of the output SOP with respect to the input frequency $\omega$. For example, if the change $\Delta \omega$ in the input frequency $\omega$ is such that the output SOP repeats itself, the change in $\delta(\omega, L)$ will be equal to $2\pi$, and hence, $\Delta \tau$ will be given by

$$\Delta \tau = \frac{2\pi}{\Delta \omega}. \quad (10.6)$$

This is the frequency domain picture of the PMD. In fact, as discussed later in this chapter, by measuring the rate of change of the output SOP with respect to the frequency (or wavelength) of input light, one can determine the PMD. This, in fact, is the working principle of several PMD measuring techniques.

It may be noted that Eq. (10.3) can also be recast as

$$\Delta \tau = L \frac{d}{d\omega} \left[ \frac{\omega}{c} (n_g - n_e) \right] = L \left[ \frac{\Delta n_{\text{eff}}}{c} + \omega \frac{d\Delta n_{\text{eff}}}{d\omega} \right]. \quad (10.7)$$

In a special case, if the dispersion of $\Delta n_{\text{eff}}$ can be neglected, then

$$\Delta \tau = \frac{L}{c} \Delta n_{\text{eff}} = \frac{\lambda_0}{c} \frac{L}{L_b}, \quad (10.8)$$

where $L_b$ represents the beat length of the fiber given by

$L_b = 2\pi / \Delta \beta = \lambda_0 / \Delta n_{\text{eff}}$.

**Example 10.1** A birefringent fiber has 1-mm beat length at $\lambda_0 = 1.5$ $\mu$m. Assuming that the difference between the effective indices of the two polarized modes is independent of the wavelength, determine the DGD in a 1-m length of the fiber at (i) $\lambda_0 = 1$ $\mu$m and (ii) $\lambda_0 = 1.5$ $\mu$m.

**Solution:** The beat length ($\lambda_0 / \Delta n_{\text{eff}}$) at $\lambda_0 = 1.5$ $\mu$m is 1 mm. This means that $\Delta n_{\text{eff}} = \frac{\lambda_0}{L_b} = 1.5 \times 10^{-3}$, and the DGD $\Delta \tau = \frac{L}{c} \Delta n_{\text{eff}} = \frac{1 \times 1.5 \times 10^{-3}}{3 \times 10^8} = 5$ ps. Since $\Delta n_{\text{eff}}$ is independent of $\lambda_0$, the DGD will be same at $\lambda_0 = 1$ $\mu$m, i.e., 5 ps.
Example 10.2 Consider a piece of high-birefringence fiber in which a polarized light beam is coupled. It is observed that if the input wavelength is changed by 1 nm, the output SOP changes from a left-circular to a right-circular SOP. Obtain the DGD in the fiber at $\lambda_0 = 1 \mu m$.

**Solution:** In a highly birefringent fiber, the DGD is given by [see Eq. (10.5)]

$$\Delta \tau = \frac{\Delta \delta}{\Delta \omega},$$

where

$$\Delta \omega = -2\pi c \frac{\Delta \lambda_0}{\lambda_0^2}.$$

Thus,

$$\Delta \tau = \frac{\Delta \delta}{\Delta \omega} = -\frac{\lambda_0^2}{2\pi c \Delta \lambda_0} \Delta \delta.$$

In the preceding three equations, $\Delta \omega$ is the change in the frequency corresponding to the wavelength change $\Delta \lambda_0$. The change in the output SOP from left circular to right circular means that $\Delta \delta = -\pi$. Substituting $\Delta \delta = -\pi$, $\Delta \lambda_0 = 10^{-9} m$, and $\lambda_0 = 10^{-6} m$, we obtain

$$\text{DGD} = \Delta \tau = \frac{10^{-12}}{2\pi \times 3 \times 10^8} \times \frac{\pi}{10^{-9}} = 1.67 \text{ps}.$$

### 10.3 PMD in Long-Length Fibers

In a long-distance fiber communication link, the fiber experiences stresses, bends, temperature changes, twists, etc., in a random fashion along the length of the link. Therefore, the birefringence along the fiber keeps changing both in magnitude as well as in direction. As a result, the birefringence is no longer additive; hence, the PMD does not grow linearly with the fiber length. Instead, it grows as a square root of the propagation distance, as will be discussed later in this section.

Due to random mode coupling, the calculation of PMD becomes very complicated. Fortunately, in a long-length fiber, there exist two orthogonal input polarization states, of which the output polarization states are independent of frequency to the first order. These input polarization states are known as the principal states of polarization (PSPs) of a given fiber length. Pulses launched in these polarization states emerge into two fixed output polarization states, which are also orthogonal to each other. Thus, if the input pulse is launched along one of the input PSPs, there is no splitting of the pulse, similar to the case with short-length or Hi-Bi fibers. The PSP model was proposed by Poole and Wagner in 1986 and is extensively used in the calculation and understanding of PMD in long-length fibers.
Polarization Mode Dispersion in Optical Fibers

It should be noted that due to random time variation of the birefringence along a long-length fiber link, PMD also varies randomly, so a statistical approach must be adopted when studying PMD. Using the coupled mode theory, Poole\(^2\) has studied the effect of random mode coupling between the PSPs on the DGD and has arrived at an elegant expression for the root-mean-square (rms) DGD given by

\[
\Delta \tau_{\text{rms}}(L) = \Delta \tau_0 \left( \sqrt{2 \frac{L_e}{L}} \right) \left( e^{-\beta' L_e} - 1 + \frac{L}{L_e} \right)^{\frac{1}{2}}, \tag{10.9}
\]

where \(\Delta \tau_{\text{rms}} = \sqrt{\langle \Delta \tau^2 \rangle}\), \(L\) is the fiber length, and \(L_e\) is a constant known as the coupling length and is a measure of magnitude of the mode coupling along the fiber length. Further, \(\Delta \tau_0\) is the DGD in the absence of mode coupling and hence will be given by Eq. (10.3), i.e., \(\Delta \tau_0 = \Delta \beta' L \).

Equation (10.9) is valid for all values of \(L\). For \(L \ll L_e\),

\[
\Delta \tau_{\text{rms}}(L) = \Delta \tau_0 \left( \sqrt{2 \frac{L_e}{L}} \right) \left( \frac{L}{\sqrt{2L_e}} \right) = \Delta \tau_0 = \Delta \beta' L, \tag{10.10}
\]

showing that the DGD varies linearly with \(L\). On the other hand, for \(L \gg L_e\),

\[
\Delta \tau_{\text{rms}}(L) = \Delta \tau_0 \left( \sqrt{2 \frac{L_e}{L}} \right) \left( \frac{L}{L_e} \right)^{\frac{1}{2}} = \Delta \beta' \sqrt{2L_L}, \tag{10.11}
\]

where we have used \(\Delta \tau_0 = \Delta \beta' L\). Equation (10.11) shows that in long-length fibers, the DGD grows as the square root of the length of the fiber. The parameter \(L_e\) is generally used to define the short-length and the long-length regimes of PMD as \(L \ll L_e\) for the short-length regime, and \(L \gg L_e\) for the long-length regime.

Equation (10.11) suggests that PMD in a long-length fiber can be reduced by reducing \(\Delta \beta'\) and \(L_e\). Reducing \(\Delta \beta'\) means that the birefringence in the fiber should be reduced, which obviously reduces PMD. Reducing \(L_e\) means that the coupling between the polarization modes should be increased. This is achieved in spun fibers, which are fabricated by spinning the fiber during the fiber-drawing process.\(^3,4\) Such fibers were first fabricated for their applications in fiber sensors. Galtarossa and coworkers\(^5,6\) have done extensive work on the PMD characteristics of spun fibers. Their studies have shown that periodic spinning in optical fibers is much more effective than uniform spinning for reducing the PMD in optical fibers.
10.4 Theoretical Modeling of PMD

As mentioned earlier, the birefringence along a long-length fiber keeps changing both in magnitude as well as in direction. As a result, the birefringence is no longer additive as in a constant-birefringence case such as a short-length fiber or a high-birefringence fiber. The PSP model, however, can be used to understand the PMD of a given long-length fiber in a similar fashion as the fixed polarization modes for a short-length or Hi-Bi fiber. Using the PSP model, Poole and coworkers have derived a dynamical equation predicting the growth of the PMD along the fiber length. We will first discuss the concept of a PMD vector and the birefringence vector, which are used in the derivation of the dynamical equation discussed in Sec. 10.4.3.

10.4.1 PMD vector

Using the PSP model, in a given length \( z \) of the fiber, the DGD can be described by an equation similar to Eq. (10.5):

\[
\Delta \tau = \frac{d}{d\omega} [z(\beta_{s}^{\text{pp}} - \beta_{l}^{\text{pp}})] = \frac{d}{d\omega} \delta^{\text{pp}}(\omega, z),
\]

where \( \delta^{\text{pp}}(\omega, z) \) represents the phase difference accumulated between the fast and slow PSPs of the fiber of length \( z \), which is responsible for the change in the output SOP as \( \omega \) changes. If the input frequency \( \omega \) is varied, \( \delta^{\text{pp}}(\omega, z) \) will also vary; hence, on the Poincaré sphere, the output SOP will rotate on a circle whose rotation axis is parallel to the slow output PSP represented by \( \hat{P}_{\text{pp}} \), as shown in Fig. 10.2. It is clear from Eq. (10.12) that the DGD is simply the rate of this rotation with \( \omega \). In view of this, the PMD of a given fiber length is generally described by a rotation vector in 3D Stokes subspace, defined as

\[
\Omega = \Delta \tau \hat{P}_{\text{pp}},
\]

where \( \Delta \tau \) is the DGD, and \( \Omega \) is known as the PMD vector whose magnitude is the DGD and whose direction is parallel to the slow output PSP in the Stokes subspace. If the input frequency is changed by \( \Delta \omega \), the change in \( \delta^{\text{pp}}(\omega, z) \) will be equal to \( \Delta \tau \Delta \omega \). Thus, the change in the SOP at \( z \) can be obtained by rotating the sphere counterclockwise around \( \Omega \) by an angle \( \Delta \tau \Delta \omega \) (see Fig. 10.2).

Let \( \hat{S}(\omega, z) \) and \( \hat{S}(\omega + \Delta \omega, z) \) represent the polarization states at distance \( z \) at frequencies \( \omega \) and \( \omega + \Delta \omega \), respectively. It is clear from Fig. 10.2, that the change \( \Delta \hat{S} \) in SOP will be such that
Figure 10.2 As frequency changes by $\Delta \omega$, the SOP at distance $z$ along the fiber will rotate from point A to B on a circle by an angle $\Delta \delta$ with rotation axis parallel to $\Omega$.

\[
\Delta \delta = \Delta \tau \Delta \omega
\]

As one moves along the fiber length, the local birefringence changes the direction as well as the magnitude of the PMD vector. In order to include the effect of the local birefringence on the PMD vector in moving from $z$ to $z + \Delta z$, we define another vector known as the birefringence vector.

10.4.2 Birefringence vector

As one moves from distance $z$ to $z + \Delta z$, the SOP will be changed due to the local birefringence. At a given frequency $\omega$, the phase difference accumulated between the two polarization modes of fiber length $\Delta z$ will be given by
Here, $\beta_s$ and $\beta_t$ represent the propagation constants of the slow and the fast polarization modes of fiber section of length $\Delta z$. The change in the SOP can thus be obtained on the Poincaré sphere by rotating it counterclockwise by an angle $\Delta \delta'(z)$ around the Stokes vector, representing the local slow polarization mode (see Section 7.4). As the pulse moves along $z$, the rate of the previously mentioned rotation is given by

$$\frac{d\delta'(z)}{dz} = (\beta_s - \beta_t) = \Delta \beta.$$  

(10.17)

Thus, at a given frequency $\omega$, the evolution of the SOP of the pulse along the fiber length can be described by a rotation vector $\beta$ (known as the birefringence vector), given by

$$\beta = \Delta \beta \hat{P}_{\text{local}},$$  

(10.18)

where $\Delta \beta = (\beta_s - \beta_t)$, and $\hat{P}_{\text{local}}$ represents the Stokes vector of the local slow polarization mode of the fiber section of length $\Delta z$. Thus, on the Poincaré sphere, $\beta$ represents a vector whose magnitude is $\Delta \beta$ and whose direction is along $\hat{P}_{\text{local}}$. Clearly, as the pulse moves along the fiber, its SOP will be moving on a circle with its rotation axis parallel to $\beta$. The change in the SOP in moving a distance $\Delta z$ along the fiber can thus be obtained from its SOP at $z$ by rotating the sphere counterclockwise around $\beta$ by an angle $\Delta \beta \Delta z$ (see Fig. 10.3).

Let $\hat{S}(\omega, z)$ and $\hat{S}(\omega, z + \Delta z)$ represent the polarization states at frequency $\omega$ at distances $z$ and $z + \Delta z$, respectively. It is evident from Fig. 10.3 that the change $\Delta \hat{S}$ in SOP will be such that

$$\left|\Delta \hat{S}\right| = \hat{S}(\omega, z + \Delta z) - \hat{S}(\omega, z) = A'B' = \Delta \beta \Delta z \sin \theta',$$

i.e.,

$$\Delta \hat{S} = \Delta z \beta \times \hat{S}(\omega, z).$$  

(10.19)

Thus, the evolution of the SOP along the fiber length at frequency $\omega$ can be described by the following differential equation:

$$\frac{\partial \hat{S}((\omega, z)}}{\partial z} = \beta \times \hat{S}(\omega, z).$$  

(10.20)
Figure 10.3 As distance changes by $\Delta z$, the SOP will rotate from point $A'$ to $B'$ on a circle by an angle $\Delta \delta'$ with rotation axis parallel to $\beta$.

It may be mentioned here that in the case of long-length fibers ($L \gg L_c$), the directions of $\Omega$ and $\beta$ are generally different, both varying with $z$. On the other hand, for a short-length fiber, since the birefringence is constant, and there is no polarization mode coupling along the fiber length, the directions of the $\Omega$ and $\beta$ vectors will be parallel to each other.

10.4.3 Dynamical equation for the PMD vector

A dynamical equation\(^7\) predicting the growth of the PMD vector along the fiber length can be derived using Eqs. (10.15) and (10.20). Assuming the continuity of the PMD vector, one can write

\[
\frac{\partial}{\partial z} \left( \frac{\partial \hat{S}(\omega, z)}{\partial \omega} \right) = \frac{\partial}{\partial \omega} \left( \frac{\partial \hat{S}(\omega, z)}{\partial z} \right). \tag{10.21}
\]

Substituting $\partial \hat{S}/\partial \omega$ from Eq. (10.15) and $\partial \hat{S}/\partial z$ from Eq. (10.20) into the preceding equation, one obtains

\[
\frac{\partial \Omega(\omega, z)}{\partial z} \times \hat{S}(\omega, z) + \Omega(\omega, z) \times \frac{\partial \hat{S}(\omega, z)}{\partial z} = \frac{\partial \beta(\omega, z)}{\partial \omega} \times \hat{S}(\omega, z) + \beta(\omega, z) \times \frac{\partial \hat{S}(\omega, z)}{\partial \omega}.
\]
Substituting again $\partial \hat{S}/\partial \omega$ and $\partial \hat{S}/\partial z$ in the preceding equation, and using the vector identity $A \times (B \times C) = B (A \cdot C) - C (A \cdot B)$, one obtains

\[
\frac{\partial \Omega(\omega, z)}{\partial z} \times \hat{S}(\omega, z) = \frac{\partial \beta(\omega, z)}{\partial \omega} \times \hat{S}(\omega, z) + [\beta(\omega, z) \times \Omega(\omega, z)] \times \hat{S}(\omega, z),
\]

i.e.,

\[
\frac{\partial \Omega(\omega, z)}{\partial z} = \frac{\partial \beta(\omega, z)}{\partial \omega} + \beta(\omega, z) \times \Omega(\omega, z).
\]

Equation (10.22) is the required dynamical equation that describes the growth of the PMD vector along the fiber length. It must be noted that the first term on the right-hand side is the local DGD per unit length, which is the deriving term responsible for the growth of PMD. The second term accounts for the change of the PMD vector due to local birefringence, because of which the DGD does not grow linearly with the fiber length.

Poole and coworkers\(^7\) have solved the preceding equation using Monte Carlo simulations, taking the random variation of birefringence into account. Their results show that the probability density function for the magnitude of the PMD in a long-length fiber follows a Maxwellian distribution,\(^7,8\) given by

\[
P(\Delta \tau) = \frac{32}{\pi^2} \frac{\Delta \tau^2}{\langle \Delta \tau^2 \rangle^3} \exp\left(- \frac{4\Delta \tau^2}{\pi \langle \Delta \tau^2 \rangle^2}\right),
\]

where $\langle \Delta \tau \rangle$ is the mean DGD.

### 10.4.4 Concatenation model: an alternative approach

An alternative approach to understanding the PMD characteristics of a long-length transmission fiber of length $L$ is to consider the given fiber as a concatenation of an $N$ number of small, linearly birefringent fiber segments whose lengths are randomly chosen around a certain mean length. Each segment has a fixed set of birefringent axes; however, various sections are oriented randomly with respect to the other sections.\(^9,10\) Figure 10.4 shows the schematic of the concatenation model.

The Jones matrix $T_n$ for the $n$th section can be written as

\[
T_n = \begin{pmatrix}
\cos \theta_n & -\sin \theta_n \\
\sin \theta_n & \cos \theta_n
\end{pmatrix} \begin{pmatrix}
\exp\left(i \frac{Bh_n}{2}\right) & 0 \\
0 & \exp(-i \frac{Bh_n}{2})
\end{pmatrix} \begin{pmatrix}
\cos \theta_n & \sin \theta_n \\
-\sin \theta_n & \cos \theta_n
\end{pmatrix},
\]

(10.24)
where $B = (\beta_1 - \beta_2)$ represents the birefringence, $\theta_n$ is the orientation of the fast axis of the $n$’th segment with respect to a fixed laboratory axis (for instance, the $x$ axis), and $h_n$ is the length of the $n$’th segment. Thus, the Jones matrix corresponding to the entire length of fiber can be expressed as a multiplication over $T_n$ for all of the $N$ sections:

$$T = \prod_{n=1}^{N} T_n .$$

To calculate the DGD, the method of Jones matrix eigenanalysis is used, which is discussed in the following section.

### 10.4.4.1 Jones matrix eigenanalysis

In this technique, we first obtain the Jones matrix $T(\omega_1)$ and $T(\omega_2)$ of the given fiber length $L$, predicting the output polarization states corresponding to a given input SOP $\mathbf{S}_i$ at two closely spaced frequencies $\omega_1$ and $\omega_2$. If $\mathbf{S}_o(\omega_1,L)$ and $\mathbf{S}_o(\omega_2,L)$ are the corresponding output polarization states,

$$\mathbf{S}_o(\omega_1,L) = T(\omega_1)\mathbf{S}_i, \quad (10.26)$$

and

$$\mathbf{S}_o(\omega_2,L) = T(\omega_2)\mathbf{S}_i. \quad (10.27)$$

Using Eqs. (10.26) and (10.27), we then obtain the matrix relating $\mathbf{S}_o(\omega_1,L)$ to $\mathbf{S}_o(\omega_2,L)$ as

$$\mathbf{S}_o(\omega_2,L) = \Gamma(\vec{\alpha},\Delta\omega) \mathbf{S}_o(\omega_1,L), \quad (10.28)$$
Chapter 10

where \( \vec{\omega} = (\omega_S + \omega_L)/2 \), \( \Delta \omega = (\omega_L - \omega_S) \), and \( \Gamma(\vec{\omega}, \Delta \omega) = T(\omega_L)T^{-1}(\omega_S) \) describes the evolution of the output SOP with frequency. The eigenstates of \( \Gamma(\vec{\omega}, \Delta \omega) \) are the two PSPs, and hence, its eigenvalues are \( \rho_s = \exp(-i \tau_s \Delta \omega) \) and \( \rho_f = \exp(-i \tau_f \Delta \omega) \), respectively, corresponding to the slow and fast eigenvectors. The DGD is then given by

\[
\Delta \tau(\vec{\omega}) = \tau_s - \tau_f = \frac{\text{Arg}(\rho_s / \rho_f)}{\Delta \omega},
\]

(10.29)

where \( \tau_s \) and \( \tau_f \) are the group delay times of the pulses traveling along the slow and fast PSPs of the fiber, and \( \text{Arg}(\ ) \) is the argument function. Here, the value of \( \Delta \omega \) is chosen such that the product \( \Delta \tau \Delta \omega \) is sufficiently small to avoid any omission of full-2\( \pi \) rotations in the \( \text{Arg}(\ ) \) function.

To obtain the probability distribution of the DGD, \( \Delta \tau \) is calculated a large number of times by selecting \( \theta_n \) to be a random quantity uniformly distributed in \([0, \pi]\) and \( h_n \) to be a random quantity chosen from a Gaussian distribution around the mean length \( h_m = L/N \) with a standard deviation \( \Delta h \). These values of \( \Delta \tau \) are arranged in the form of a relative frequency distribution, which is then fitted to a Maxwellian distribution of the form

\[
P(\Delta \tau) = \frac{32}{\pi^2} \frac{\Delta \tau^2}{\langle \Delta \tau \rangle^3} \exp\left(-\frac{4\Delta \tau^2}{\pi\langle \Delta \tau \rangle^2}\right),
\]

(10.30)

where \( \langle \Delta \tau \rangle \) is the mean DGD. The mean PMD is then calculated as

\[
D_{\text{PMD}} = \frac{\langle \Delta \tau \rangle}{\sqrt{L}}.
\]

(10.31)

10.5 PMD Measuring Techniques

The PMD of a given device is completely known if we know its polarization vector \( \Omega(\omega) \) as a function of frequency \( \omega \). As discussed earlier, the magnitude of \( \Omega(\omega) \) is equal to the DGD of the device, and its direction is the direction of the slow output PSP represented by the vector \( \hat{P}_{psp} \) in the 3D Stokes subspace. Thus, the measurement of PMD is basically the measurement of the PMD vector \( \Omega(\omega) \). The measurement techniques can broadly be classified as (i) time-domain techniques and (ii) frequency-domain techniques.

A particular technique can be considered to be a time-domain technique provided \( \tau_c < \Delta \tau \) and as a frequency-domain technique if \( \tau_c \gg \Delta \tau \), where \( \tau_c \)}
represents the coherence time of the light used in the measurement and $\Delta \tau$ is the DGD of the device under test (DUT). Various PMD measurement techniques are discussed in a recent book by Galtarossa and Menyuk.\textsuperscript{12} In the following two sections, we discuss the working principle of these techniques briefly.

### 10.5.1 Time-domain techniques

The most simple and intuitive time-domain technique involves measuring $\tau_s$ and $\tau_f$ directly and calculating DGD as $\tau_s - \tau_f$. The corresponding measurement apparatus is also simple in principle and is shown schematically in Fig. 10.5. Short, polarized pulses are launched into the DUT, and the times of arrival of the two output pulses traveling along the slow and fast PSPs are measured using a fast oscilloscope. The input SOP is changed continuously using a polarization controller, and when it matches either of the input PSPs, only one output pulse will emerge. By measuring the polarization state of the slow output pulse, one can also obtain the $\hat{P}_{psp}$. One important requirement of this technique is that the width of the input pulses be extremely small in order to achieve the desired DGD resolution.

### 10.5.2 Frequency-domain techniques

The working principle of various frequency-domain techniques uses the fact that as the input frequency is changed, the change in the output SOP is described by

$$\frac{d\hat{S}(L,\omega)}{d\omega} = \Omega \times \hat{S}(L,\omega). \quad (10.32)$$

As discussed earlier, the preceding equation means that as the input frequency changes, the output SOP $\hat{S}(L,\omega)$ rotates on the Poincaré sphere, around the PMD vector $\Omega$. The rate of this rotation is given by

$$\left| \frac{d\delta}{d\omega} \right| = |\Omega| = \Delta \tau = \text{DGD}. \quad (10.33)$$

![Figure 10.5](http://ebooks.spiedigitallibrary.org/01212016TermsOfUse.aspx) Schematic experimental setup used to measure PMD using time-domain techniques.
In Eq. (10.33), $d\delta$ represents the change in the phase difference accumulated between the output PSPs of the DUT due to a change in frequency $d\omega$. Thus, all frequency-domain techniques measure the variation in the output SOP with respect to the input frequency. The various frequency-domain techniques are thus basically polarimetric techniques. The schematic experimental setup in all of such techniques is similar and is shown in Fig. 10.6. Continuous and polarized light from a tunable laser with an appropriate polarization state (so that light is coupled in both of the PSPs) controlled by a polarization controller is passed through the DUT. At the output end, the SOP is measured as a function of input frequency using a polarimeter.

The experimental data obtained using frequency-domain techniques can be used to obtain the PMD vector $\Omega$ by one of the following methods:

(i) Jones matrix eigenanalysis (JME),
(ii) Mueller matrix method (MMM), or
(iii) Poincaré sphere analysis (PSA).

Accordingly, frequency-domain techniques are also classified as JME, MMM, and PSA methods. The JME method was briefly discussed in Sec. 10.4.4.1; the working principles of the other two methods are similar.12

There are other methods that do not use JME, MMM, or PSA methods. In the next section, we discuss a polarimetric technique that is extremely simple to implement. This scheme measures only the DGD and is known as the wavelength-scanning method or the fixed-analyzer method.13

10.5.3 Wavelength-scanning method

The output SOP of a device with fixed eigen SOPs, such as a length of a high-birefringence fiber, traces a circle on the Poincaré sphere as the wavelength is changed. The axis of rotation will be parallel to the slow polarization mode $\hat{P}_s$ of the fiber [Fig. 10.7(a)]. However, in a more general case, the birefringence may be due to a variety of factors—stress, twists, bends, etc.—whose direction and magnitude may change randomly along the fiber length and with time. In such a case, the output SOP wanders on the sphere in a random fashion, as shown in Fig. 10.7(b).

The changes in the output polarization state are transformed into variations in the output light intensity by putting a “fixed analyzer” at the output, followed by an optical power meter. A schematic setup for the wavelength-scanning method for the measurement of PMD is shown in Fig. 10.8.

![Figure 10.6 Schematic experimental setup used to measure PMD using frequency-domain techniques.](http://ebooks.spiedigitallibrary.org/)
It is easy to understand that in the case of a Hi-Bi fiber, the output intensity will vary periodically (as shown in Fig. 10.9) with wavelength, and each period will correspond to the change in the phase difference between the two polarization modes as $\Delta \delta = 2\pi$. Thus, by counting the number of times the intensity crosses the mean level, one can determine the DGD. For example, if the number of full cycles in the intensity variation between wavelengths $\lambda_1$ and $\lambda_2$ is $N$, then

$$
\Delta \delta = 2N\pi, \text{ and } \Delta \omega = c \frac{2\pi}{\lambda_1 - \lambda_2} \left( \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right).
$$

(10.34)

Using Eq. (10.5),

$$
\Delta \tau = \frac{\Delta \delta}{\Delta \omega} = \frac{N \lambda_1 \lambda_2}{c (\lambda_2 - \lambda_1)}.
$$

(10.35)
In the case of a Hi-Bi or a short-length fiber, there is no polarization mode coupling, and the output intensity variation is periodic with a fixed amplitude and period, as discussed earlier. However, in long-length single-mode fibers, due to random variation in the birefringence and finite coupling between orthogonal polarization states, the output intensity variation looks somewhat like that shown in Fig. 10.10.

**Figure 10.9** Schematic representation of the variation in output intensity with respect to wavelength in a wavelength-scanning technique for a Hi-Bi or short-length fiber.

**Figure 10.10** Schematic representation of the variation in output intensity with respect to wavelength in a wavelength-scanning technique for a long-length single-mode fiber.
By counting the number of oscillations in intensity about a mean threshold level, the value of DGD can still be estimated and is given by

$$\Delta \tau = \frac{N k \lambda_1 \lambda_2}{c (\lambda_2 - \lambda_1) \tau},$$  

where constant $k$ depends on the mode coupling, and $k = 0.81$ is recommended for highly mode-coupled fibers. For standard communication-grade fibers, the PMD is typically 0.01 to 0.1 ps/km. It may be mentioned that, due to mode coupling, PMD in long lengths of fibers is proportional to the square root of the fiber length.

### 10.6 PMD Mitigation Techniques

Polarization-induced impairments are the main factor limiting the bit-rate per channel in a long-haul communication system. Furthermore, because of its statistical nature, one cannot fix an upper limit on the penalty imposed due to the PMD. Whatever limit one fixes, there is always a finite amount of probability (known as the outage probability) that the PMD-induced penalty will exceed the limit. The maximum tolerable limit of PMD is considered to be 10 to 20% of the bit duration. In an ultra-long-haul communication system, this limit is readily reached, due to the presence of chromatic dispersion compensators and optical amplifier modules. Thus, mitigating the PMD-induced impairments is extremely important. In this regard, research efforts are focused on two different goals: (i) producing low-PMD fibers and (ii) developing PMD compensators. In the following sections, we briefly discuss these two aspects of PMD mitigation.

#### 10.6.1 Low-PMD fibers

As discussed earlier, an ideal single-mode fiber does not have any birefringence and hence, no PMD. Real fibers, however, have some core ellipticity as well as asymmetrical stresses present in the core that make the fiber birefringent, both of which are sources of PMD. Since it is extremely difficult to completely eliminate this birefringence, one has to find ways of reducing its contribution toward PMD. Research efforts made in this direction during the last two decades have established that spinning the fiber at very high speed while it is in the molten stage can significantly reduce the PMD. It should be understood that spin is different from twist. Twist is introduced in the cold fiber, producing torsional stress and introducing a circular birefringence in the fiber through the photoelastic effect. On the other hand, spinning the fiber in the molten state introduces only the rotation of the birefringence axes of the fiber, without introducing any torsional stress or circular birefringence in the fiber.

The working principle of PMD reduction through spinning can be understood by considering the fiber as various birefringent sections of equal length but rotated with respect to each other. Consider the case in which each section is
rotated by an angle of $\pi/2$ with respect to the previous section. Now, the fast (slow) axis of each section will be parallel to the slow (fast) axis of the following section. As a result, the PMD accumulated in one section will be exactly compensated for in the next section.

Both unidirectional as well as periodic spin functions are reported to obtain low-PMD fibers.\textsuperscript{17-19} In order to obtain the exact factor by which the PMD is reduced due to spin, one has to solve the dynamical equation (10.21). In the case of unidirectional spin, Schuh et al.\textsuperscript{17} have reported the analytical solution for the dynamical equation and have shown that due to the spin, the PMD is reduced by a spin-induced reduction factor (SIRF), given by

$$\text{SIRF} \approx \frac{1}{\sqrt{1 + 4 \left( \frac{L_b}{p} \right)^2}}, \quad (10.37)$$

where $L_b$ represents the beat length of the unspun fiber, and $p$ is the spin period, i.e., the fiber length over which the fiber completes one rotation. To gain a sense of the numerical value of the SIRF, if we consider $L_b = 10$ m, and $p = 1$ m, then SIRF $\approx 0.05$.

\textbf{10.6.2 PMD compensators}

The second approach to PMD mitigation is to use a PMD compensator. It should be mentioned that it is extremely difficult to completely compensate for PMD, as it is a complex phenomenon due to its randomly varying nature. In order to understand the basic concepts involved in PMD-compensation techniques, we first discuss what is known as first- and second-order PMD.

\textbf{10.6.2.1 First- and second-order PMD}

PMD of a fiber can be described by the PMD vector $\Omega$, which is given by $\Omega = \Delta \tau \hat{P}_{\text{pp}}$ [see Eq. (10.13)], and which is frequency dependent. If we perform a Taylor series expansion of $\Omega(\omega)$ around the carrier frequency $\omega_c$, we obtain

$$\Omega(\omega_c + \Delta \omega) = \Omega(\omega_c) + \Omega'(\omega_c) \Delta \omega + \ldots, \quad (10.38)$$

where

$$\Omega'(\omega_c) = \left. \frac{d\Omega(\omega)}{d\omega} \right|_{\omega_c}. \quad$$

In the preceding equation, the first and second terms on the right-hand side are known as the first- and second-order PMD, respectively.\textsuperscript{20} There will be higher-
order terms as well, which are known as higher-order PMDs. It should be noted that the first-order PMD is frequency independent.

The second-order PMD is proportional to $\Omega'(\omega_0)$ and can be written as

$$
\frac{d\Omega(\omega)}{d\omega} \bigg|_{\omega_0} = \frac{d(\Delta \tau)}{d\omega} \bigg|_{\omega_0} \hat{P}_{pp}^*(\omega_0) + \Delta \tau(\omega_0) \frac{d\hat{P}_{pp}}{d\omega} \bigg|_{\omega_0}.
$$

Thus, the second-order PMD consists of two terms: the first term is parallel to $\hat{P}_{pp}^*(\omega_0)$, i.e., $\Omega(\omega_0)$, and the second term is perpendicular to $\Omega(\omega_0)$, since $d\hat{P}_{pp}/d\omega$ is perpendicular to $\hat{P}_{pp}$.

Neglecting higher orders of PMD, Fig. 10.11 schematically represents the PMD vector $\Omega(\omega)$ and its first- and second-order PMD components, where vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ represent the first- and second-order PMD components of the total PMD vector $\overrightarrow{AC}$, while vectors $\overrightarrow{BD}$ and $\overrightarrow{DC}$, respectively, represent the component of the second-order PMD that is parallel and perpendicular to the first-order PMD.

Various schemes are reported in the literature for compensating the first- and second-order PMD components. First-order PMD compensators are relatively easy to realize, the basic working principle of which is discussed briefly in the next section.

**Figure 10.11** Schematic representation of the PMD vector $\Omega(\omega)$ and its first- and second-order PMD components.
10.6.2.2 Working principle of a first-order PMD compensator

It should be clear from the preceding discussion that to the first-order approximation, PMD splits the input pulse into two pulses polarized along fast and slow PSPs. At the output end, these pulses are separated by a time delay that is equal to the DGD of the fiber. Both the directions of the PSPs as well as the magnitude of the DGD keep changing with time.

It must also be understood that for a given value of DGD, the PMD-induced impairments will be maximized when both PSPs carry equal power. For example, if only one PSP is excited, there will be no PMD-induced signal distortion. Similarly, if the DGD is equal to zero, i.e., both pulses reach the output end at the same time, again there will be no PMD-induced signal distortion. On the Poincaré sphere, this means that if the input polarization state $\hat{S}_i$ is parallel to either $\omega$ or $\Omega = 0$, there will be no PMD-induced impairments, which is what a first-order PMD compensator is designed to achieve. Accordingly, there are two points of operation of a first-order PMD compensator, as discussed in the remainder of the section.

A PMD compensator consists of a Hi-Bi fiber, a polarization controller (PC); a detector measuring the quality of the signal, (e.g., DOP), and a control algorithm that sends a feedback signal to the PC, as shown in Fig. 10.12. The compensator module is inserted at the receiver end of the fiber link. If the PMD of the fiber link and that of the compensator are denoted by $\Omega_L$ and $\Omega_C$, respectively, the overall PMD of the system (including the compensator) $\Omega_{tot}$ will be given by $\Omega_{tot} = \Omega_L + \Omega_C$ (see Fig. 10.13).

After inserting the compensator in the link, the PMD-induced impairments will be decided by the overall DGD = $|\Omega_{tot}|$ and the angle $\theta_{tot}$ between $\hat{S}_i$ (input SOP) and the $\Omega_{tot}$ (overall PMD) shown in Fig. 10.13. By making either the overall DGD or the angle $\theta_{tot}$ equal to zero, the PMD of the fiber link can be completely compensated. These two points of operation of a PMD compensator are shown in Fig. 10.14 and defined here:
Fig. 10.13 Schematic vector representation of the PMD of the fiber link $\Omega_L$, the compensator $\Omega_C$, and the total PMD $\Omega_{\text{Tot}}$ with respect to the input polarization state $\hat{S}_i$.

Figure 10.14 Two points of operation of a first-order PMD compensator: (a) $\Omega_{\text{Tot}} = 0$ and (b) $\Omega_{\text{Tot}}$ is parallel to the input polarization state $\hat{S}_i$.

(i) $\Omega_{\text{Tot}} = 0$: It is easy to understand that $\Omega_{\text{Tot}}$ will be zero if $\Omega_C = -\Omega_L$. However, since $\Omega_L$ varies randomly with time and $\Omega_C$ is fixed, this point of operation cannot be achieved perfectly.

(ii) $\theta_{\text{Tot}} = 0$: This point of operation when $\Omega_{\text{Tot}}$ is parallel to $\hat{S}_i$ can be rigorously achieved by selecting the value of $|\Omega_C|$ sufficiently large that $|\Omega_C| \geq |\Omega_L|$. It is easy to understand that in such a case, the resultant PMD vector $\Omega_{\text{Tot}}$ can always be made parallel to $\hat{S}_i$ so that $\theta_{\text{Tot}} = 0$.

The control algorithm is a crucial component of a PMD compensator, as it keeps tracking the local maxima in the signal quality by sending the feedback signal to the polarization controller. In the literature, various qualities of the output signal are used for generating the feedback signal; among these studies,
the DOP is most commonly used. It may be mentioned that all of the time or frequency components of the input pulse have the same polarization states, and hence, the DOP of the input pulse will be 1. However, because the input pulse splits into two pulses that travel with different group velocities, the various components of the pulse at the output end will be different. For example, the SOP of the leading edge, middle edge, and trailing edge of the output pulse will be same as the SOP of the fast PSP, input pulse, and slow PSP, respectively. As a result, the DOP of the output pulse will be degraded and will be less than 1. The controlling algorithm attempts to maximize the DOP of the output pulse by making either \( \Omega_{\text{Tot}} = 0 \) or \( \theta_{\text{Tot}} = 0 \), thus minimizing PMD-induced impairments.

References


