Polarization Aberrations in Astronomical Telescopes: The Point Spread Function

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Message

• Polarization provides
  – Important exoplanet science

• Need to model and compensate for instrument induced polarization to maximize the return from high contrast exoplanet coronagraphs
  – Science that does not require polarization
  – Polarization science
Why is polarization of interest?

- ExoPlanet polarization measurements
  - Orbital elements
  - Atmospheric composition & chemistry
  - Surface (solid & liquid)
  - Dust and gas & formation in protoplanetary systems

- Telescope & instrument internal polarization
  - Measurement errors for both polarized and unpolarized sources
  - Reduces contrast & SNR
  - Complicates calibration
For 50 years we have been calibrating telescopes for photo-polarimetry

• To measure $\{I,Q,U,V\}$ of objects—stars, interstellar matter, planets, nebulae, galaxies, quasars, etc.

• In this presentation we will show that polarization also plays a major role in high-contrast image formation
Image formation

• Review scalar wave image formation
• Vector (polarization) wave image formation
• Polarization ray trace a minimally complicated optical system
• Look at the PSF & coronagraphs
Review scalar wave image formation

\[ I_{\text{image}}(x,y) = I_{\text{Object}}(x,y) \otimes PSF \]

\[ PSF = \text{Airy diffraction pattern} = \left( \frac{2J_1(\rho)}{\rho} \right)^2 \]

If \( I_{\text{Object}}(x,y) = \delta(x,y) \) then \( I_{\text{image}}(x,y) = PSF \)

Requires ideal conditions of
No geometric aberrations &
No polarization aberrations

Over the field of view
Geometric aberrations

Wavefront error \( W \) = \( \frac{\text{reference ray path} - \text{ray path}}{\lambda} \) = \( \frac{\text{OPD}}{\lambda} \)

For all points \( x, y \) across the exit pupil

In Space, with no atmosphere, we can come close to

\[ W(x, y) = 0 \]

But . . .

\[ W(x, y) = 0 \neq \text{perfect image} \]

Need to examine polarization aberrations
Source of polarization aberrations

- Astronomical telescopes require metal mirrors for broad-band high reflectivity
- Reflection from metal mirrors => Both amplitude and phase change & the reflected ray is partially polarized
Polarization determines image quality.

E & M fields from regions A and B need to be correlated (the SAME polarization state) to form the pixels in an image.

Models that use vector representation of fields are necessary.

More later=>
Modern astronomical telescopes --

- Partially polarize the wavefront as it propagates through the telescope and instrument to the focal plane
- This physical optics phenomenon modifies the shape of the PSF
- The coronagraph focal plane mask size and its properties should be designed to match the telescope Point Spread Function (PSF)
Aberrations

It is easy to visualize surface OPD geometric wavefronts.

Challenge to visualize polarization aberrations.

HST primary 1982
8 years before launch
Propagate the **field** through the system to find the **complex scalar field** at the focal plane \( U_3(x_3, y_3) \)

\[
U_1(x_1, y_1) = \delta(x_1, y_1)
\]

\[
\begin{align*}
\tau_2(\xi_2, \eta_2) &= A_2(\xi_2, \eta_2) + i\phi_2(\xi_2, \eta_2) \\
I_3(x_3, y_3) &= |U_3(x_3, y_3)|^2
\end{align*}
\]

Where \( \tau_2(\xi_2, \eta_2) = A_2(\xi_2, \eta_2) + i\phi_2(\xi_2, \eta_2) \)
Proof: polarization role in image formation

For zero OPD error $W(x,y)=0.0$

Exit pupil

No Polarizer

Resolution is position angle independent

To represent internal polarization in the extreme we add two perpendicular linear polarizers

Resolution is position angle dependent

The PSF is the incoherent sum of two “D” apertures

Image plane PSF
Observations

• Orthogonally polarized light does not interfere to contribute to an image.
• The shape of the point spread function depends on how polarization changes across the exit pupil.

Questions?

• What are the sources of instrument polarization in astronomical telescopes?
• What is the magnitude of the effect?
• What is the impact?
Vector wave image formation

\[ \vec{U}_3(x_3,y_3) = \]
\[ K \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\vec{U}_2^{-}(\xi_2,\eta_2)] \cdot \vec{\tau}_2(\xi_2,\eta_2) \exp \left\{-j \frac{2\pi}{\lambda f} (x_3,\xi_2 + y_3,\eta_2) \right\} d\xi_2 d\eta_2 \right] \]

In astronomical telescopes and instruments the term \( \vec{\tau}_2(\xi_2,\eta_2) \) is a vector and \( \vec{U}_3(x_3,y_3) \) depends on BOTH the polarization properties of the source & the telescope/instrument.

<table>
<thead>
<tr>
<th>Pupil transmittance complex</th>
<th>Jones vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{\tau}_2(\xi_2,\eta_2) = )</td>
<td>( \begin{bmatrix} J_{XX} &amp; J_{XY} \ J_{YX} &amp; J_{YY} \end{bmatrix} )</td>
</tr>
</tbody>
</table>

Where \( J_{xx} \) is \( \vec{X} \) light in \( \vec{X} \) light out and \( J_{xy} \) is the \( \vec{X} \) light in that has been projected into \( \vec{Y} \).
Fresnel (1823) equations & definitions

\( N_0 = 1. \)

For metals, index is complex:
\[
N_1 = n_1 - ik_1
\]

\[
\theta_1 = \arccos \left( \frac{\sqrt{N_1^2 - N_0^2 \sin^2 \theta_0}}{N_1} \right)
\]

\[
r_p = \frac{\tan(\theta_0 - \theta_1)}{\tan(\theta_0 + \theta_1)}
\]

\[
r_s = \frac{-\sin(\theta_0 - \theta_1)}{\sin(\theta_0 + \theta_1)}
\]

\[
\frac{r_s - r_p}{r_s + r_p} = \text{diattenuation}
\]

\[
\tan(\psi) = \tan(\phi_s - \phi_p) = \left| r_p \right| / \left| r_s \right|
\]

\( \psi \) is called retardance
Reflection coefficients (A & $\phi$) for Al @ 800 nm; $N_1 = 2.80 + 8.45i$

The two polarization aberrations are

$$\frac{r_s - r_p}{r_s + r_p} = \text{diattenuation and retardance} \quad (\tan \psi = \frac{|r_p|}{|r_s|})$$
Decompose white-light (star) into polarization components

• We select any orthonormal basis set for ray trace
• Select the easiest for intuition
• Component perpendicular (⊥, or Y or p)
• Component parallel (∥, or X, or s)

White-light source

\[
\begin{align*}
s_{ray} &= A_X e^{-i\phi_X} \\
p_{ray} &= A_Y e^{-i\phi_Y}
\end{align*}
\]

Focal plane

\[
\begin{align*}
A_{XX} e^{-i\phi_{XX}} \\
A_{XY} e^{-i\phi_{XY}} \\
A_{YY} e^{-i\phi_{YY}} \\
A_{YX} e^{-i\phi_{YX}}
\end{align*}
\]
Polarization cross talk is increased by a change of the Eigenstate of the propagating wavefront; tilted mirrors.

All angles 90-degrees => then the Eigenstate of the final wavefront are mixed \( \perp \) and \( \parallel \).

Mirror D now sends beam into a compound angle and the cross product terms increase.
Polarization ray trace a 3-element minimally complicated (no A/R coat, one fold) layout.

We will find that

\[ I(x,y) \text{ is the sum of 4 PSF's} \]

2.4 meter F# = 1.2 aluminum coated mirrors & F# = 8 focus

Curvatures on the primary and secondary optimized for \( W(x,y) = 0 \).

To design an optimum mask for exoplanets =>
model the focal plane electric field accurately.
Polarization depends on incidence angle

\[ \theta_0 = f(\rho) \]

The incident rays march across the pupil strike the mirror at different angles, depending on radius.

Incident rays march across the fold mirror striking at decreasing angles from the top down.
\[
\frac{r_s(\xi, \eta) - r_p(\xi, \eta)}{r_s(\xi, \eta) + r_p(\xi, \eta)}
\]
diattenuation face-on surface maps

Primary M. + Secondary M. = Fold M. + Telescope

Primary is F/# = 1.2 and the F/# at the focal plane is 8.

Length of the line & orientation shows the vector of the diattenuation

Exit pupil
\[
\tan(\psi(\xi,\eta)) = \tan(\phi_S(\xi,\eta) - \phi_P(\xi,\eta))
\]
retardance face-on surface maps

Primary M. + Secondary M. + Telescope =

Primary is F/# = 1.2 and the F/# at the focal plane is 8.

Length of the line & orientation shows the vector of the retardance

Exit pupil
How to calculate the PSF for each polarization

Based on the direction cosine at each surface and the physical properties of each surface \((n - ik)\), we use the Fresnel equations to calculate the amplitude change and the phase change for each ray at each surface.

Compute the multiplicative amplitude and cumulative phase for both the \(\parallel\) and the \(\perp\) light for each ray traced across the entrance pupil & map these 2 complex arrays onto the exit pupil.
Map & group the functions

Amplitude normalized

Radians of phase

\[
\begin{align*}
A_{XX} e^{i\phi_{XX}} & \quad & A_{XY} e^{i\phi_{XY}} \\
A_{YX} e^{i\phi_{YX}} & \quad & A_{YY} e^{i\phi_{YY}} \\
\end{align*}
\]

\[
\equiv
\begin{pmatrix}
J_{XX} \\
J_{YX}
\end{pmatrix}
\]

\[
\equiv J_{\text{ExitPupil}}
\]
Learn from previous chart

- The orthogonally polarized components contain different wavefront aberrations, which differ by approximately 32 milliwaves.
- A single A/O system cannot correct for both polarizations simultaneously.
- Wedge between the two gives .6 milli arc seconds shear.

s ray is 9% brighter than the p ray
How do we calculate the PSF?

- The electric field at the focal plane is given by

\[ U_3(x_3, y_3) = \]

\[ K \begin{bmatrix}
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} J_{XX} J_{XY} \\
J_{YX} J_{YY}
\end{bmatrix} \exp\left\{-j \frac{2\pi}{\lambda f} (x_3, \xi_2 + y_3, \eta_2)\right\} d\xi_2 d\eta_2 \]

And the focal plane intensities are given by

\[ I_3(x_3, y_3) = \left| U_3(x_3, y_3) \right|^2 = \]

\[ \left| \mathcal{F}(J_{XX}) \right|^2 + \left| \mathcal{F}(J_{YY}) \right|^2 + \left| \mathcal{F}(J_{YX}) \right|^2 + \left| \mathcal{F}(J_{XY}) \right|^2 \]

The telescope PSF is the linear (uncorrelated) superposition of these 4 PSF’s.
Propagate the exit pupil field to map the 4 independent co-propagating PSF’s to the image plane.

Amplitude Response Matrix $\mathbf{ARM} = \begin{bmatrix} \ImaginaryPart[J_{XX}(x, y)] & \ImaginaryPart[J_{XY}(x, y)] \\ \ImaginaryPart[J_{YX}(x, y)] & \ImaginaryPart[J_{YY}(x, y)] \end{bmatrix}$

What does the focal plane look like?

\[
\begin{bmatrix} \ImaginaryPart[J_{XX}(x, y)]^2 & \ImaginaryPart[J_{XY}(x, y)]^2 \\ \ImaginaryPart[J_{YX}(x, y)]^2 & \ImaginaryPart[J_{YY}(x, y)]^2 \end{bmatrix}
\]
Polarization PSF ($I_{XX}$) & the “ghost” PSF ($I_{YY}$) for the 2.4 meter telescope - note the “zeros” do not line up

log_{10} Irradiance

Face-on ghost PSF
Polarization reflectivity anisotropy => changes in polarization across wavefront surface

Flavio Horowitz, 1983 & Smith/Purcell 1953

• Anisotropy is produced by the coating processes used for large telescope mirrors

\[ \vec{E} \text{ incident sees a different conductivity in the substrate depending on whether the wave is reflecting from an amorphous or the columnar structure} \]

**Side view of reflecting film**

Left columnar (crystal)
Right amorphous micro-structure

\[
\begin{align*}
\nabla \cdot D &= \rho \\
\nabla \cdot B &= 0 \\
\n\nabla \times E &= -\frac{\partial B}{\partial t} \\
\n\nabla \times H &= J + \frac{\partial D}{\partial t}
\end{align*}
\]
Summary for this telescope

- 32 milli-waves difference in the wavefront aberrations (tilt, coma, astigmatism, spherical, etc.) between $\parallel$ and $\perp$
- Shift between the PSF’s for X and Y is 0.625 masec
- X and Y show a 9% difference in intensity reflectance
• Light coupled from one polarization forms a separate faint and much larger PSF not superposed on $J_{XX}$ and the $J_{YY}$

• => complex field may spill over the edges of a mask that is designed assuming scalar diffraction.

  – Radius of 90% encircled energy:

  \[ r_{XX} = r_{YY} = 0.15 \text{ arcsec and} \]

  \[ r_{XY} = r_{YX} = 0.36 \text{ arcsec} \]
Summary for this telescope

• Unpolarized sources exit partially polarized into an instrument.

• The telescope coatings cause polarization variations throughout the PSF, particularly into the diffraction rings to complicate polarization measurements of exoplanets and debris rings in coronagraphs.
Thank you

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Method to mitigate polarization cross-talk . . . . . . =>
Linear polarizers do not mitigate these effects!

- A Wollaston beam splitter (WBS) prism placed over the focal plane does not unscramble the co-propagating mixed polarized signals.
  
  ─ They were mixed up stream in the optical path

- Since the beams are deviated in a Wollaston, the Eigenstates are projected onto a rotated coordinate system & the power in the off-diagonal elements is increased.
Several ways to mitigate these effects
One is to build a phase plate

\[ J_{T+Cgph} = \begin{pmatrix} J_{XX} & J_{YX} \\ J_{XY} & J_{YY} \end{pmatrix} \equiv \begin{pmatrix} A_{XX} e^{i\phi_{XX}} & A_{YX} e^{i\phi_{YX}} \\ A_{XY} e^{i\phi_{XY}} & A_{YY} e^{i\phi_{YY}} \end{pmatrix} \]

To minimize the polarization effects, we need to develop a corrective optical element whose Jones pupil, \( J_{\text{corrector}} \), has the property:

\[ J_{\text{System}} = \left( J_{T+Cgph} \right) \cdot \left( J_{\text{Corrector}} \right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]
Mitigation: spatially variable retarder plate

Figure 4. Photo alignment layer of a SVRP plate. (a) shows a spatially variable retarder plate (SVRP) face on (x,y) with retardance direction indicated by the colors shown in the stripe below which maps color into orientation in degrees as shown. (b) shows two particular regions, P1 and P2 which have two different polarization states and orientation are shown. (c) shows a diagram of the typical stack or sandwich. The bottom layer is a layer of homogeneous dielectric oriented to the polarization direction. Birefringent layers of B1, B2...Bn will be deposited with thickness layers and specific process recipe calculated and optimized to compensate for the Fresnel polarization of light reflected form the telescope.
The end
New work

• Measure the polarization reflectivity anisotropy and its spatial scale on a large astronomical telescope mirror
• Select a practical coronagraph design and calculate contrast using vector wavefronts
• Refine models to calculate vector diffraction around masks and stops
• Once we have contrast $= f($polarization$)$, then search for practical mitigation approaches
Why current test beds see very high contrast?

• Ground test beds have few to no fold mirrors & high F#'s, unlike many space and ground-based telescope systems
• Our work was at 800nm. N+iK depends on wavelength
• Test beds function at narrow bandwidths
• Industry uses proprietary dielectric overcoated metal mirrors – we used no overcoat
New work

• Develop a coronagraph test bed that emulates a practical system, measure the polarization aberrations and validate the models
• Explore a spatially variable wave plate which will correct “as-built” telescope systems.
• How much internal polarization can we have and still achieve the $10^{-11}$ extinction needed for terrestrial exoplanets?
• Determine the requirements on the physical properties of the surfaces, # of mirrors, angles, masks, transmittance, etc.
• Design and develop masks and stops to optimize terrestrial exoplanet characterization in the presence of polarization aberrations
Curve B is the diffraction pattern from holes P1 and P2

- Spacing of the fringes underneath curve B is related to the separation of the holes P₁ and P₂
- Visibility (contrast) of these fringes underneath curve B is given by the degree of correlation (coherence) of the fluctuating electromagnetic fields between P₁ and P₂.
• If the light from $P_1$ is polarized orthogonal (linear or circular) to light from $P_2$
  
  – Fringes under the dotted curve disappear
  
  – Light from $P_1$ is no longer correlated with light from $P_2$
    
    • $P_1$ and $P_2$ both remain independent white light sources
    
    • The intensity on the screen is the LINEAR superposition of the patterns