Cladding effects in single-mode fiber: space and polarization phenomena

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ABSTRACT

The work presents results of the measurement of spatial and polarization far-field characteristics of the output radiation from the double-clad fiber. The beam width is found to depend on the input conditions. Polarization characteristics of the beam show existence of spatially-incoherent fields in the fiber which cause depolarization of the total output field. Theoretical interpretation of the effects is done in the framework of the modal approach. By means of numerical simulations is shown that far-field output beam width, as well as it's degree of polarization, depends on the input gaussian beam spatial parameters due to interference between core mode and cladding modes of the fiber.

Keywords: optical fibers, fiber-optic interferometry, cladding modes, degree of polarization, Stocks parameters, input conditions.

1. INTRODUCTION

Light beam which falls upon the input end of waveguide is known to be accepted and guided by the waveguide core only if the light beam has appropriate spatial parameters. To excite the waveguide modes effectively we need to match the input spatial distribution with the guided mode one. If the waveguide is excited by unmatched beam the power losses become inevitable when the field of input beam is transformed into the field of guided modes: source spherical wave and leaky modes radiate first, then only guided modes propagate within the fiber on the large distance from the input. However real waveguides have finite cladding and some part of the radiated power is reflected from the surface between inner cladding and protector. Depending on the indices of refraction of the core \( n_{co} \), cladding \( n_{cl} \) and protector \( n_p \) the step index double-clad waveguides are known to be of a low-index inner cladding \( (n_{co} > n_p > n_{cl}) \) and of a high-index inner cladding \( (n_{co} > n_{cl} > n_p) \). The first ones support only leaky modes, the second ones support leaky and guided modes. Cladding modes may appear in the real waveguide not only due to unmatched input, but due to scattering of radiation on the material defects of the waveguide, due to unadiabatic field transformation in fiber-optic elements, in splices and connectors, etc. As a result of cladding modes and core mode interference the output beam intensity distribution changes, as well as the beam state of polarization.

As well known, the modes of the circular dielectric waveguide are hybrid, their electric and magnetic field directions depending on the transverse coordinate. In weakly-guiding fiber polarization effects are small, fields...
of the modes are described by the scalar wave equation solutions and may be presented by uniformly-polarized LP-modes. Therefore weakly-guiding fiber with circular core spatially homogeneous along it's axis doesn't vary polarization characteristics of the propagating radiation. If the weakly-guiding fiber has small nonuniformities randomly arranged along the fiber axis, the phases of different polarization modes become stochastic.\(^3\)\(^4\) As a result direction of the total field vector occurs to depend on the transverse coordinate, providing depolarization of the output light beam.

In the work we consider a single-mode double-clad weakly-guiding fiber. Cladding modes are treated as the solutions of scalar wave equation for multimode waveguide consisting from the inner cladding and protector. Below are presented: 1)the results of experimental investigation of the far-field spatial and polarization characteristics of the output light beam; 2)mathematical model of the double-clad fiber and analysis of the modes excitation depending on the input conditions; 3) theoretical study of the light beam state of polarization in the double-clad fiber; 4) calculation of the Stocks parameters of the total field in the fiber with random nonuniformities; 5) results of the numerical simulations of the core mode and cladding modes interference (the fiber was assumed to be excited by the single-frequency gaussian beam).

2. EXPERIMENT

The experimental setup is described in detail in the Ref.5. A linearly polarized \(TEM_{00}\) \(He-N\)laser with 2-4 longitudinal modes was used as the light source (\(\lambda = 0.63\mu m\)). The input power was controlled and scanned by a combination of rotatable phase plates. The state of polarization of the output radiation was measured by the same device. In measurements we used the single-mode fiber with silicon rubber protector (length of the fiber 10cm, core diameter 8\(\mu m\),cladding diameter 125\(\mu m\), \(n_{co} - n_{cl} \approx 0.001\)). \(He-N\)laser radiation was launched into the input end of the fiber by means of microobjective. Photodetector placed on the rotatable device enabled to scan far-field output intensity distribution with the step of 15' (approximately 60 points on the spot dimension).

To analyze the obtained curves of intensity spatial distribution (without polarizers) we have used the method of moments.\(^6\) The \(n\)-order moment is \(M_n = \sum_{k=1}^{N} x_k^n f(x_k)\), where \(f(x_k)\)-is a power measured in the point \(k\). As an approximation of the measured power distribution we have used gaussian function \(f(x) = I_0 \exp[-2(r - a)^2/w^2]\). The beam width \(w\) was determined as \(w = 2(\mu_2 - \mu_1)^{1/2}\), where \(\mu_n = M_n/M_0\) are the normalized moments. To control the measurement accuracy we have calculated the parameter \(\kappa = \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3\) which is of zero value in the case of gaussian distribution.\(^6\) In our experiment the obtained values \(\kappa = \pm 0.01\) show the gaussian approximation to be adequate. However experimental points which present the field intensity in the narrow region (\(\approx 60\)) near the center of the spot lie consistently higher than the gaussian curve, plotted according to the results of the experimental data processing. Changing the gaussian beam width on the input end of the fiber with microobjective, we have obtained the far-field spot width \(w_f\) dependence on the moment \(M_0\) (Fig.1,solid line), which is proportional to the effectiveness of the core mode excitation. The beam width decreasing when the input conditions become worse indicates excitation of fields with lower angular divergence comparing with the core mode divergence. Then appearance of the hump in the center of the far-field beam spot may be accounted for by the low-divergent cladding modes distribution (see the picture in the frame in the Fig.1).

To investigate output polarization characteristics the polarization modulators of the setup were used. Mueller matrix measurements in each point of the beam cross-section were performed by the technique presented in Ref.5. As a result of the experiments the Mueller matrix elements \(M_{ij}\) were found to depend on the transverse coordinate. Moreover it appeared that \((\sum_{ij} M_{ij}^2)/M_{11}^2 < 4\), the inequality indicating depolarization of the output radiation.\(^7\)

Thus we have found that not only core mode is excited in the single-mode fiber, but spatially incoherent fields with low angular divergence may be excited, too. The fields may be treated as guided or leaky cladding modes. The cladding modes and core mode interference appreciably changes spatial and polarization characteristics of the output radiation.
3. THEORY

As a model of double-clad fiber we use coaxial structure, consisting from the core of radius \( a \), cladding of the radius \( b \) and infinite protector. Since the core and cladding indices of refraction differ slightly, the fiber may be considered\(^2\) as weakly-guiding structure which supports modes with uniform linear polarization (LP-modes). Transverse fields of the modes are defined\(^8\) as Bessel functions: \( J_n(u_{nm}r/a), J_n(v_{nm}r/a), Y_n(v_{nm}r/a) \) and \( K_n(w_{nm}r/a) \), where \( u_{nm}, v_{nm}, w_{nm} \) - are the transverse wavenumbers in core, cladding and protector, respectively.

It is well known that protector only slightly changes the core modes of weakly-guiding waveguide with thick inner cladding \( b \gg a \). Characteristic equation of the structure transforms into the characteristic equation of waveguide with infinite cladding.\(^1\)

Let us consider, otherwise, how the single-mode core changes the guided modes of multimode structure with inner cladding as a core and protector as a cladding. Transverse distribution of \( LP_{nm} \)-modes of the structure can be written as

\[
E_x = \begin{cases} 
AJ_n(v_{nm}r/b), & r \leq a \\
CJ_n(u_{nm}r/b) - DY_n(u_{nm}r/b), & a < r \leq b \\
FK_n(w_{nm}r/b), & r > b 
\end{cases}
\]

Through the simple transformation we obtain the characteristic equation for the cladding modes:

\[
\frac{u_{nm}J_{n+1}(u_{nm})}{J_n(u_{nm})} - \frac{w_{nm}K_{n+1}(w_{nm})}{K_n(w_{nm})} = \frac{J_{n+1}(cu_{nm})Y_n(w_{nm})}{J_n(u_{nm})Y_{n+1}(w_{nm})} \times \frac{[J_n'(cu_{nm}) - J_n'(cw_{nm})]Y'_n(u_{nm}) - K'_n(w_{nm})}{Y'_n(cw_{nm}) - J'_n(cw_{nm})}.
\]

Here \( c = a/b \), \( f'_n(p) = f_n(p)/[p f_{n+1}(p)] \).

If the core is narrow \( (c \approx 0) \), right part of the Eq.(2) becomes zero. Then Eq.(2) transforms into characteristic equation of the waveguide with infinite cladding (see Eq.(4.103) from the Ref.1). Thus if \( a \ll b \) the core doesn’t influence on the cladding modes. For \( LP_{0m} \)-modes of the waveguide with the parameter values \( c = 0.1, V_{cl} = \)
2\pi b(n_c^2 - n_i^2)1/2/\lambda = 17, V_{co} = 2\pi a(n_c^2 - n_i^2)1/2/\lambda = 1.7 we have obtained \delta u_{nm} = u_{nm} - u_{nm} \approx 10^{-2}, \delta \beta_{nm} = \beta_{nm} - \beta_{nm} \approx 10^{-6}. Here u_{nm}, \beta_{nm} - wavenumbers obtained by numerical solution of the Eq.(2); u_{nm}, \beta_{nm} - of the Eq. (4.103) from the Ref.1. Therefore if a \ll b it is possible to consider the total field of the double-clad fiber to be a sum of the field of the waveguide consisting from the core and infinite inner cladding and the field of multimode waveguide consisting from the inner cladding and infinite protector.

2. The field of the gaussian beam which falls upon the input end of the fiber is:

\[ E^i = E_0 \exp \left( -\frac{i}{w_g} \left( (x - x_0)^2 + (y - y_0)^2 \right) \right), \]

where \(w_g\) is the beam width, \(R\) - radius of phase front curvature; \(x_0, y_0\) - coordinates of the gaussian beam transverse displacement; \(k_0\) - wavenumber.

To begin with analysis of the double-clad fiber excitation we note that cladding modes form a set of orthogonal functions which are not orthogonal to the core mode function. Therefore from the condition of transverse fields continuity on the input end of the fiber\(^1,2\) one obtains an intricate system of linear equations. If \(b \gg a\) and gaussian beam is narrow \((w_g/a \approx 1)\), the calculations may be simplified as follows: first the gaussian beam is assumed to excite the field of the fiber consisting from the core and inner cladding, the field being a sum of core mode \(E^{co}\) and field of radiation. The latter then partially transforms into the field of the cladding modes \(E^{cl}\). Then on the input end of the fiber

\[ E^i = E^{co}, \]

\[ E^{co} = E^{rad}, \]

where

\[ E^{co} = (A_0^x \bar{e}_x + A_0^y \bar{e}_y) E_0^{co}(r) e^{i\delta_0z} + E^{rad}, \]

\[ E^{cl} = \sum_{n = -N}^{N} \sum_{m = 1}^{M} (a_{nm}^x \bar{e}_x + a_{nm}^y \bar{e}_y) E^{cl}_{nm}(r) e^{i\delta_{nm}z} + E^{rad}. \]

Here \(A_0^x, A_0^y\) - \(a_{nm}^x, a_{nm}^y\) are complex amplitudes of the core mode and cladding modes of two orthogonal polarizations; \(\bar{e}_x, \bar{e}_y\) - unit vectors of the \(x, y\) - axes; \(N, M\) - maximum values of the \(n, m\) - mode indices; \(E_0^{co}(r), E_0^{cl}(r)\) - radial distributions of the core mode and cladding modes:

\[ E_0^{co}(r) = \begin{cases} J_0(w_0r/a)/J_0(w_0), & r \leq a \\ K_0(w_0r/a)/K_0(w_0), & r > a \end{cases} \]

\[ E_{nm}^{cl}(r) = \begin{cases} J_n(u_{nm}r/b)/J_n(u_{nm}), & r \leq b \\ K_n(u_{nm}r/b)/K_n(u_{nm}), & r > b \end{cases} \]

The field of radiation \(E^{rad}\) has radial power distribution \(I(r)\) which differs from the gaussian one\(^2\). But we have shown for the narrow light beams \((w_g \ll b)\) that the power distribution between the cladding modes slightly depends on the form of \(I(r)\) function. In practice usually the optimal conditions of core mode excitation realize. Then below we shall consider only narrow beams \((w_g/a \leq 3)\) with small displacement relative to the fiber axis \((x_0, y_0 \leq w_g)\). Then to calculate the amplitude coefficients \(A_0, a_{nm}\) we can use the proper expressions from the Ref.9.

Let us consider for simplicity gaussian beam which falls onto the input end of the fiber parallel to it's axis, the plane of the beam waist coinciding with the plane of the fiber end surface. Transverse displacement of the beam is considered to realize along one of the reference axes (we take \(x\) - axis).

Input power distribution between the cladding modes is uneven and depends on the input conditions. In the Fig.2 the power spectra of the cladding modes are shown, with some values of gaussian beam width \(w_g\) and
displacement $x_0$. The multimode fiber with $V_2 = 17$ (corresponding number of guided modes $N = 12$) was taken into consideration. Here horizontal are azimuthal indices $-12 \leq n \leq 12$ and vertical are radial indices $m$ of the modes. Evidently narrow beams excite dominantly cladding modes with the less caustic radii (since $r_c/b \simeq n/u_{nm}^2$). When the beam displaces in transverse direction the power redistributes between the modes with the larger radii of caustics and basic mode, which provide narrow far-field structure.

$w_g/a = 1.$

$x_0/a = 0.$

$w_g/a = 2.$

$x_0/a = 0.$

1.

2.

Figure 2: power spectra of the fiber modes.

Effectiveness of $LP_{01}$ - mode excitation versus the value of $w_g/a$ has a maximum$^{1,2}$ which corresponds to the optimal input conditions ($w_g = w_{opt}$). In the Fig.3a we have plotted core mode power $P^{co}$ dependence on the width of gaussian beam of unit power, using the expression (4). The same curves were plotted for the power of total field of cladding modes $P^{cl}$, singly the multimode waveguide was taken into consideration. We have used the expression (4a) assuming $E_{rad}^{co} = E^1$, where $E^1$ - field of gaussian beam of unit power. Solid lines are obtained for input beam without displacement, dashed lines - with displacement $x_0/a = 1$. Evidently, in the case of multimode fiber all input power is launched into the guided modes, excluding the range of narrow beams ($w_g/a \leq 2$).

In order to calculate amplitudes of the modes we need to know how the input power distributes between the core and cladding. Since we are dealing with the narrow beams, to obtain the core mode power $P^{co}$ we can use the expression (4), the total power of the cladding modes being $P^{cl} = (1 - P^{co})P_1^{cl}$, where $P^{cl}$ - power of the cladding modes, calculated for the gaussian beam of unit power. In the Fig.3b values of $P^{cl}$ and $P^{co}$ are presented versus the input conditions ($w_g$ changes, $x_0 = 0$).

3. In scalar approximation far-field and near-field spatial distributions are connected by the Fraunhofer integral$^{10}$.
According to the integral form of Bessel functions:

\[ J_n(x) = \frac{1}{2\pi} \int_0^{2\pi} d\varphi \exp(iz\sin\varphi \cos(\varphi - \varphi_0)) \],

we can estimate integrals on \( \varphi \) in (7):

\[ \frac{1}{2\pi} \int_0^{2\pi} d\varphi \exp(iz\cos(\varphi - \varphi_0)) = r^n \exp(\pm in\varphi)J_n(z). \]

After integration on \( r \) and \( \varphi \) in (7) we obtain:

\[ E_{z,y}(z_c, z, \psi) = A_{x,y}^0 \frac{V^2}{(u_0^2 - z_c^2)(w_0^2 + z_c^2)} \left[ \frac{u_0J_1(u_0)}{J_0(u_0)} - z_cJ_0(z_c) \right] e^{i\beta_{z}L} + \]

\[ + \sum_{n=-N}^{N} \sum_{m=1}^{M} \alpha_{nm} e^{i\psi} \left[ \frac{u_{nm}J_{n+1}(u_{nm})}{J_n(u_{nm})} - z_cJ_{n+1}(z_c) \right] e^{i\beta_{nm}L}. \]

The summands in (10) corresponds to the far-field angular distribution: the first - of the core mode; the second - of the total field of the cladding modes.

4. The pairs of modes with orthogonal polarization \( E_{nm}^x \) and \( E_{nm}^y \) acquire some different random phases \( (\varphi_{nm}^x, \varphi_{nm}^y) \) when the modes propagate in the fiber with random nonuniformities:

\[ E_{nm}^z = |E_{nm}^x|f_{nm}(r, \varphi)\exp(i\varphi_{nm}^x)\exp(i\beta_{nm}z). \]
Here \( f_{nm}(r, \varphi) \)-spatial distribution of the mode in transverse cross-section of the waveguide. As a result of the mode interference vector of the total field

\[
\vec{E} = \sum_{n=-N}^{N} \sum_{m=0}^{M} \vec{E}_{nm} = E_x \vec{e}_x + E_y \vec{e}_y
\]  

(12)

describes an ellipse of polarization in any point of the cross-section, the ellipse shape and orientation depending on the transverse coordinate. Local Stocks vector \( S(r, \varphi, z) \) in any point of the fiber cross-section is given by the parameters:

\[
\begin{align*}
I(r, \varphi, z) &= E_x E_x^* + E_y E_y^*, \\
Q(r, \varphi, z) &= E_x E_y^* - E_y E_x^*, \\
U(r, \varphi, z) &= E_x E_x^* + E_y E_y^*, \\
V(r, \varphi, z) &= i(E_x E_y^* - E_y E_x^*),
\end{align*}
\]  

(13)

which satisfy the relation \( I^2 = Q^2 + U^2 + V^2 \) for completely polarized radiation. Integration of (13) over the transverse cross-section of the fiber provides the integral Stocks vector:

\[
S(z) = \int_{S_d} S(r, \varphi, z) dS,
\]  

(14)

where \( S_d \) is area of integration. Since fiber modes are orthogonal in any infinite cross-section,\(^2\) if \( (S_d \rightarrow \infty) \) the integral Stocks parameters are constant and doesn’t depend on \( z \):

\[
\begin{align*}
I_N &= \sum_{nm} (|E_{nm}^x|^2 + |E_{nm}^y|^2) N_{nm}, \\
Q_N &= \sum_{nm} (|E_{nm}^x|^2 - |E_{nm}^y|^2) N_{nm}, \\
U_N &= \sum_{nm} |E_{nm}^x||E_{nm}^y| \cos(\varphi_{nm} - \varphi_{nm}^x) N_{nm}, \\
V_N &= \sum_{nm} |E_{nm}^x||E_{nm}^y| \sin(\varphi_{nm} - \varphi_{nm}^y) N_{nm}.
\end{align*}
\]  

(15)

where \( N_{nm} = \int \int |f_{nm}|^2 (r, \varphi) r dr d\varphi \). If the fiber is uniform along its axis the integral degree of polarization

\[
P_N = \sqrt{Q_N^2 + U_N^2 + V_N^2} / I_N,
\]  

(16)

doesn’t vary when the light beam propagates through the fiber.

When random fluctuations in the fiber material exist the Stocks parameters vary randomly following random changes of the phases \( \varphi_{nm}, \varphi_{nm}^x, \varphi_{nm}^y \).

We have shown above (see expression (10)) that Fraunhofer diffraction doesn’t disturb near-field mode azimuthal distribution, but changes the radial one. Then the far-field modes become non-orthogonal in infinite cross-section. Consequently the far-field integral Stocks parameters appear to depend on the fiber length \( L \):

\[
\begin{align*}
I_F &= \sum_{n,m} (|E_{nm}^x|^2 + |E_{nm}^y|^2) F_{nm} + \sum_{n,m} \sum_{m_1 \neq m_1} (|E_{nm}^x||E_{nm_1}^x| \exp(i\Delta \varphi_{nm_1}) + \\
&+ |E_{nm}^y||E_{nm_1}^y| \exp(i\Delta \varphi_{nm_1}) F_{nm_1} \exp(i\Delta \varphi_{nm_1})), \\
Q_F &= \sum_{n,m} (|E_{nm}^x|^2 - |E_{nm}^y|^2) F_{nm} + \sum_{n,m} \sum_{m_1 \neq m_1} (|E_{nm}^x||E_{nm_1}^x| \exp(i\Delta \varphi_{nm_1}) - \\
&- |E_{nm}^y||E_{nm_1}^y| \exp(i\Delta \varphi_{nm_1}) F_{nm_1} \exp(i\Delta \varphi_{nm_1})), \\
U_F &= \sum_{nm} |E_{nm}^x||E_{nm}^y| F_{nm} \cos \Delta \varphi_{nm} + \sum_{n,m} \sum_{m_1 \neq m_1} (|E_{nm}^x||E_{nm_1}^y| \exp(i\Delta \varphi_{nm_1} + \\
&+ |E_{nm}^y||E_{nm_1}^y| \exp(i\Delta \varphi_{nm_1}))
\end{align*}
\]  

(17)
\[ + |E^y_{nm}|^2 |E^x_{nm}| \exp(-i\Delta \varphi_{nm}) F_{nm1} \exp(i\Delta \beta_{nm1} z), \]
\[ V_F = \sum_{nm} |E^x_{nm}|^2 |E^y_{nm}| F_{nm} \sin \Delta \varphi_{nm} + \sum_{nm} \sum_{m_1 \neq m} (|E^x_{nm}|^2 |E^y_{nm}| \exp(i\Delta \varphi_{nm}) - |E^y_{nm}|^2 \exp(i\Delta \varphi_{nm}) F_{nm1} \exp(i\Delta \beta_{nm1} z). \]

Here \( \Delta \varphi_{nm} = \varphi^x_{nm} - \varphi^y_{nm} ; \Delta \varphi_{nm} = \varphi^y_{nm} - \varphi^x_{nm} ; \Delta \beta_{nm1} = \beta_{nm} - \beta_{nm1} ; \)
\[ F_{nm} = \int \int |f_{nm}|^2 (\rho, \psi) d\rho d\psi ; F_{nm1} = \int \int |f_{nm}(\rho, \psi)||f_{nm1}(\rho, \psi)| d\rho d\psi, \]
where \( f_{nm}(\rho, \psi) \)-far-field transverse distribution of the mode.

5. Integral near-field Stocks parameters of the total cladding and core field given by the expressions (5), can be written as follows
\[ I = (|A_0^x|^2 + |A_0^y|^2) J_0 + \sum_{nm} (|a_{nm}^x|^2 + |a_{nm}^y|^2) J_{nm}, \]
\[ Q = (|A_0^x|^2 - |A_0^y|^2) J_0 + \sum_{nm} (|a_{nm}^x|^2 - |a_{nm}^y|^2) J_{nm}, \]
\[ U = 2 |A_0^x| |A_0^y| \cos \Delta \varphi_0 J_0 + 2 \sum_{nm} |a_{nm}^x| |a_{nm}^y| \cos \Delta \varphi_{nm} J_{nm}, \]
\[ V = 2 |A_0^x| |A_0^y| \sin \Delta \varphi_0 J_0 + 2 \sum_{nm} |a_{nm}^x| |a_{nm}^y| \sin \Delta \varphi_{nm} J_{nm}, \]
\[ + \sum_{nm} \sum_{m_1 \neq m} (|a_{nm}^x| |a_{nm}^y| \exp(i\Delta \varphi_{nm}) - |a_{nm}^y| |a_{nm}^x| \exp(-i\Delta \varphi_{nm}) J_{nm1} \exp(i\Delta \beta_{nm1} z). \]

Here \( \Delta \varphi = \varphi^x - \varphi^y ; \) The parameters \( J_0, J_{nm}, J_{nm1} \) are the integrals of combinations of the functions (6),(6a) over the cross-section \( S_d \) of the fiber. When the cross-section is infinite we obtain \( J_{nm1} = 0 \). Then the Stocks parameters and degree of polarization don’t depend on \( z \). If \( S_d < \pi b^2 \), we have \( J_{nm1} \neq 0 \) and the Stocks parameters vary along the fiber axis. The far-field Stocks parameters are given by the same expressions as (18), but \( J_0, J_{nm}, J_{nm1} \) present the integrals of the functions (10) over the cross-section of the fiber, the integrals \( J_{nm1} \) always being nonzero.

### 4. NUMERICAL SIMULATIONS

1. Far-field intensity distribution of the total field of core and cladding modes is given by the expression (10), which is too complicated to study analytically the far-field spot size dependence on the input conditions. We have performed computer simulations, considering the input gaussian beam to be parallel to the fiber axis, the beam waist coinciding with the input end surface. We have used the proper expressions from the Ref.9 to calculate
the amplitude coefficients in (10). The core mode and cladding modes far-field interference was considered in the plane located at some distance from the output end of the fiber. With the values \( n_{co} - n_{el} = n_{el} - n_p = 0.001 \), \( c = 10 \), \( \lambda = 1 \mu \) we have obtained \( V_1 = 1.7, V_2 = 17 \). Accounting for spatial and polarization degeneration we had the number of cladding modes \( N_2 = 126 \). The random phases \( \varphi_{nm}, \psi_{nm} \) quantities were generated by the built-in computer function.

![Fig. 4](http://example.com/fig4.png)

**Figure 4:** Narrowing of the output beam due to cladding modes contribution.

Far-field radial intensity distribution \( |E|^2 \) of the core mode is well approximated by a gaussian function with the width \( w_{co}^2 \), which doesn't depend on the input conditions (Fig.4a, dashed line). As a result of core mode and cladding modes interference a narrow structure appears at the top of the curve (Fig.4, solid line). Since far-field envelope of radial distribution of the cladding modes, excited by nondisplaced gaussian beam is satisfactorily approximated by the gaussian curve (\( k \approx 0.001 \)), we can use the method of moments to estimate the width of cladding modes intensity distribution \( w_{F}^2 \). In order to compare the results with the experimental ones we have plotted \( w_{F}^2 \) dependence on the effectiveness of the core mode excitation \( P_{co}/P_{co}^{opt} \) (Fig.4b), where \( P_{co}^{opt} \) is the core mode power obtained with the optimal input conditions (\( w_g = w_{opt} \)). The part I of the curve is plotted with \( w_g < w_{opt} \), the part II - with \( w_g > w_{opt} \). Evidently when input conditions are optimal the far-field spot size of the total field is the greatest and equal to the core mode spot size \( w_{co}^2 \). Otherwise the output spot size becomes less due to contribution of the cladding modes. Note that input power distributes between the cladding modes so that wide input gaussian beam provides output far-field radiation with small angular divergence which forms narrow far-field structure.

2. We have calculated local near-field Stocks parameters (13) on the output end of the fiber. For simplicity we have supposed the core mode to be linearly polarized in any cross-section of the fiber (\( \varphi_0^p = \varphi_0^n, |A_0^p| = |A_0^n| \)), the normalized Stocks parameters being the follows: \( I = 1, Q = 0, U = 1, V = 0 \). The both polarization \( LP_{nm} \) cladding modes were assumed to be excited with equal amplitudes (\( |a_{nm}^x| = |a_{nm}^y| \)). With the values \( V_1 = 1.7, V_2 = 17 \) we have plotted (Fig.5): near-field \((I_N)\) and far-field \((I_F)\) radial intensity distribution of the total output beam;
Figure 5.

$$w_z/a = 1., \ x_0/a = 0.$$
near-field and far-field degree of linear polarization \( P^l_N = \sqrt{Q^2_N + U^2_N/I_N} \), \( P^f_N = \sqrt{Q^2_F + U^2_F/I_F} \), near-field and far-field degree of circular polarization \( P^c_N = \sqrt{V^2_N/I_N} \), \( P^c_F = \sqrt{V^2_F/I_F} \) in transverse cross-section of the output light beam. If only core mode would be excited in the fiber, the output light beam would be linearly polarized: \( P^l_N = P^f_N = 1 \), \( P^c_N = P^c_F = 0 \). The cladding modes excitation leads to disturbance of the uniform character of the beam polarization.

![Figure 6: polarization of the output beam depending on the input conditions.](image)

The integral degree of output beam polarization \( (P_N, P_F) \) is plotted in the Fig.6 depending on the input gaussian beam width \( w_g \) \((x_0 = 0)\). Near-field integral Stocks parameters were calculated with the expressions (18) assuming \( S_d \to \infty \). Far-field integral Stocks parameters were obtained by numerical integration of the local Stocks parameters (13) over the total cross-section of the fiber. The curve 1 is obtained for the total field of core mode and cladding modes, the curve 2 - only for the cladding modes. When optimal input conditions realize \( (w_g = w_{opt}) \) the degree of total field polarization is the greatest indicating that input gaussian beam completely transforms into the guided core mode. Otherwise the cladding modes contribution increases leading to depolarization.

We see from the Fig.6, that small variations of the input gaussian beam width cause the degree of output beam polarization to be changed too. The effect is pronounced when conditions are close to the optimal ones \( (w_g \approx w_{opt}) \). In the region \( w_g/a > 2 \) degree of output beam polarization slightly depends on \( w_g \). Then to obtain more stable output polarization characteristics of a fiber-optic device it is more preferable to excite the fiber with gaussian beam of the width which is greater than the optimal one \( (w_g/a \approx 2) \).

The integral degree of cladding modes (without core mode) polarization averaged in infinite cross-section seems to be almost constant and changes slightly due to Fraunhofer diffraction (Fig.6, curves 2). However the far-field degree of the total light beam polarization is always greater than the near-field one due to the inversion of the spatial intensity distribution of the core mode and cladding modes fields when they transform from the near- to far-field regions.
5. SUMMARY

The experimental results and numerical simulations indicate that the double-clad fiber supports not only core mode, but the fields with lower angular output divergence (cladding modes) appear to be guided, too. As a result of the core mode and cladding modes interference a narrow structure appears in the center of the far-field output beam spot, the beam width depending on the input conditions. When the input conditions are optimal the beam width is the greatest and is equal to the output beam width of fiber with infinite cladding. Otherwise the far-field output light beam becomes more narrow due to the cladding modes contribution. Moreover, when the core mode of the fiber interferes with the cladding modes within the fiber with random nonuniformities, depolarization of the output light beam occurs because polarization distribution of the total field becomes nonuniform.

We have to note that in the work only guided cladding modes of the double-clad fiber with high-index inner cladding were studied. However taking into consideration leaky modes of double-clad fiber we would receive the analogous results. Small differences would appear because the leaky mode excitation alters from the guided mode one.

As a result of cladding modes excitation in real fiber-optic devices an additional noise component appears which depends on the stability of input unit and other elements.

6. REFERENCES